Large-N Chern-Simons Models in String and M-Theory

A Synopsis submitted in partial fulfillment of the requirements for the award of the degree of

Doctor of Philosophy

in

Physics and Computer Science

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March 2014

1 Introduction

Quantum field theory (QFT) is a particularly accurate and useful framework for studying quantum systems consisting of many particles, whose number may or may not be conserved [1, 2, 3]. The classic example widely quoted as an evidence for high accuracy of quantum electrodynamics (which is a quantum field theory) is prediction of the anomalous magnetic moment of the electron; which has been verified by experiment to an accuracy of better than 10 parts per billion [4]. These predictions are based on perturbative calculations in the fine structure constant, which are only possible because quantum electrodynamics is weakly interacting. Studying strongly interacting quantum field theories in theoretical physics is still a major challenge.

1.1 Groups and Symmetries in Quantum Field Theories

Quantum Field Theory involves the conceptual framework of group theory. So, an understanding of the basics of group theory is essential.

Groups are used to precisely describe symmetry in the theory and thereby simplify the understanding and calculations of physical quantities of the subject. In group theory, two types of groups are possible: discrete and continuous. Discrete groups, for instance, may involve a symmetry operation on a crystal which leaves the crystal invariant. Continuous groups involve rotation and Lorentz groups, which depend on the continuously varying parameter, namely, the angle of rotation. It is the continuous groups, known as the Lie groups which are of most importance to us [2, 3]. Broadly, there are three types of symmetries that appear in quantum field theories:

1. Internal Symmetries: Mixing of Quarks of approximately same mass is an example of an internal symmetry of the system. So, this symmetry mixes particles having a common property. These symmetries can be local or global (i.e., independent of space-time). They form compact groups, for example the rotation group, where the parameter, the angle of rotation can vary from 0 to 2π, including 2π.

2. Space-Time Symmetries: Lorentz and Poincare groups are the prominent examples. They are non-compact, i.e., the range of their parameters exclude the end points.

3. Supersymmetry: Combines Space-Time and Internal symmetries. In a supersymmetric theory, for every fermion there exists a corresponding boson (super-symmetric partner of fermion) and vice-versa.

A familiar group in three dimensions is the $SO(3)$ group, which has the following generators:

$$J_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad J_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad J_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \tag{1}$$

It is a non-abelian group, that describes rotations in three dimensions. A rotation about, say +ve z $-$ axis by angle θ is given by:

$$R_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

The group $SO(N)$ can be thought of as a generalization of the $SO(3)$ group, where the $SO(N)$ group contains $N \times N$ orthogonal matrices, and the determinant of each component matrix is unity. The condition for orthogonality is $O^T\ O\ =\ I$.

Similarly,the group $SU(2)$ group consists of 2×2 matrices which are unitary, with unit determinant, i.e. $UU^\dagger = 1$ and $\det(U) = 1$ (which leads to "special" unitary matrices of order 2). In the same manner, SU(N) group consists of $N \times N$ matrices which are unitary, with unit determinant, i.e. $UU^\dagger = 1$ and $\det(U) = 1$. There is a correspondence in between $SU(2)$ transformations in spinor space and an $O(3)$ transformation in the Euclidean space $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

The Lorentz group is one of the most important groups from the quantum field theory perspective. The most general Lorentz transformation is composed of three generators for boosts in three directions and three generators for rotation about the three axis, which relate two inertial frames moving with a relative velocity with respect to each other. While pure rotations form a group; the subset of only pure boosts do not form a group [3].

We can form a generalization of the Lorentz group by adding translations. The Poincare group is the Lorentz group with translations. This is a 10-parameter group consisting of 4-translations, 3-rotations and 3-boosts. In addition to the usual Lorentz group generators, a new translation generator needs to be added to form the Poincare group [3].

The conformal group is a generalization of the Poincare group. The generators of the conformal group include the dilatations(or rescaling) D, special conformal transformations K_μ, generators of Lorentz group $M_{\mu\nu}$ and the translations, P_μ. (The special conformal translations are inversion followed by a translation and then again an inversion.)

2 Conformal Field Theories

Polyakov's classic 1970 paper [5] marked the birth of conformal field theory where he noticed that Onsager's solution to the 2-d Ising model possessed conformal invariance at the critical point.

Many properties of a system close to a continuous phase transition turn out to be largely independent of the microscopic details of the system. Instead, they fall into one of a relatively small number of different classes, which are described as *conformal field theories* (CFT's)[6]. Correlation functions and other physical observables in CFT's are constrained due to the fact that the dynamics of the theory is invariant under conformal transformations. See, e.g., [7, 8].

Physical quantities in quantum field theory are given by correlation functions. Analogous to the expectation value of x^2 in simple harmonic oscillator, which relates to fluctuations in the system $\Delta x^2 = \langle x^2 \rangle - \langle x \rangle^2$, the correlation functions quantify the range of fluctuations in a specific quantity at two different times or position in space.

In non-conformal quantum field theories the fluctuations die out as $\langle O(x)O(y) \rangle \sim e^{-\frac{x-y}{L}}$, where L is the correlation length. But in conformal theories, the fluctuations must obey a power law $\langle O(x)O(y) \rangle \sim \frac{1}{(x-y)^\delta}$, because there is no length scale in a conformal field theory.

Equivalently, one could say that, in conformal field theories, correlation lengths diverge.

By dimensional analysis, the exponent δ is related to the dimensions of the observable $O(x)$ whose fluctuations we are studying. It turns out that δ depends on the strength of interactions, parameterized by a coupling constant λ. A typical example, can be $\delta = 4 + \frac{4\lambda^2}{3} + O(\lambda^4)$ for a particular operator. This implies that the dimensions of the observable also depend on the strength of interactions. And these are known as anomalous dimensions.

Conformal field theories in two dimensions have been well studied and show a rich structure. Here we will be interested in three dimensional conformal field theories.

3 Chern-Simons Theories

Another possible action for a spin-1 particle in 2+1 dimensions, apart from the Maxwell's action is the Chern-Simons action. It is known to arise in effective field theory description of quantum Hall systems. The Chern-Simons term arises only in 3-dimensional space-time.

The Chern-Simons theory is a mathematical way to describe the behavior of anyons. On exchanging any two anyons, they pick up a phase value, which may lie between the phase values of bosons and fermions. In general for anyons,

$$\psi(x_1, x_2) = \exp^{i\theta} \psi(x_2, x_1) \tag{3}$$

where θ varies between 0 to 2π.

Anyons can be classified into abelian and non-abelian anyons. Abelian anyons are known to play a major role in fractional quantum Hall effect [9]. The possibility of anyon high temperature superconductors has also been researched [10].

Anyons are possible only in three dimensions (2+1) and physically can be thought of as arising due to Ahranov-Bohm effect. From the application point of view, for building a topological quantum computer, particles like non-abelian anyons, whose world lines cross over to form braids in a three dimensional space-time are a necessity. Unlike abelian anyons, non-abelian anyons have not yet been detected, so the topological quantum computer is just a theoretical construct at the moment.

The Chern-Simons kinetic term for the gauge field is:

$$S_{CS} = -\frac{k}{4\pi} \int \text{tr} \left(A \wedge dA + \frac{2i}{3} A \wedge A \wedge A \right) . \tag{4}$$

where the integer k represents the Chern-Simons level [11, 12]. Chern-Simons theories naturally arise in condensed matter physics as effective field theories for quantum Hall systems [9, 13].

Early studies of Chern-Simons theories with matter include [14, 15, 16]. Supersymmetric Chern-Simons theories with matter [17, 18, 19, 20, 21, 22, 23, 24, 25] have been and continue to be the subject of intense study in string theory. They exhibit various non-trivial dualities, in addition to the AdS/CFT correspondence. See e.g., [26, 27].

From literature review one can note that conformal field theories in three dimensions without supersymmetry; which are more physically relevant compared to the theories with supersymmetry, have not been explored until recently. These Chern-Simons theories with matter, can be studied in a large-N expansion, as exact solutions as in case of theories with supersymmetry may not be possible.

4 Large N expansion

Quantum Chromodynamics (QCD) is based on gauge group $SU(3)$. In general studying the dynamics of theories based on gauge group $SU(N)$ is hard, but calculations in the quantum theory simplify on taking the limit N large. Introducing a large number N – typically the rank of a gauge group or a global symmetry group $U(N)$ or $O(N)$ – suppresses fluctuations by the law of large numbers.

The large N expansion has a long history in quantum field theory and statistical physics [28, 29, 30, 31]. There exist two very different classes of large-N theories. One class, exemplified by the $O(N)$ vector model, contains only degrees of freedom that transform as N-dimensional vectors under $O(N)$ symmetry transformations. Interactions in the large N limit are naturally parameterized by a coupling constant called the 't Hooft coupling, which one can denote by λ. In the large-N limit, these theories can generically be solved to all orders in perturbation theory in the 't Hooft coupling by a variety of techniques.

A second class of large N theories arise when one considers gauge theories. Gauge fields, by definition, transform in the adjoint representation of the gauge group and are therefore $N \times N$ matrices rather than N dimensional vectors. Unlike vector models, even in the large N limit, these theories cannot be solved to all orders in perturbation theory. Before the AdS/CFT correspondence, the nature of the strongly interacting large N saddle point of these theories was a complete mystery.

't Hooft observed that, in the large N limit, only planar Feynman diagrams, the ones which can be drawn on the surface of a plane – contribute to physical quantities. The corrections in $1/N$ come from non-planar diagrams drawn on surfaces of higher genus, much like the sum over worldsheets in string theory.

5 Physical Motivations

Subir Sachdev, a Harvard condensed matter physicist writes, "A significant part of modern physics research can be classified as the study of quantum matter. Its aim is to describe the phases of large numbers of interacting particles at temperatures low enough so that quantum mechanics play a crucial role in determining the distinguishing characteristics" [32].

The most common phases of electrons known to us are − metals, superconductors and insulators. But this classification does not take into account spin associated with an electron. Correlated electron materials or strongly interacting materials are the subject of much recent theoretical and experimental investigation. Unlike the metals where the electron-electron repulsion is neglected compared to the kinetic energy of the electrons, in correlated systems this

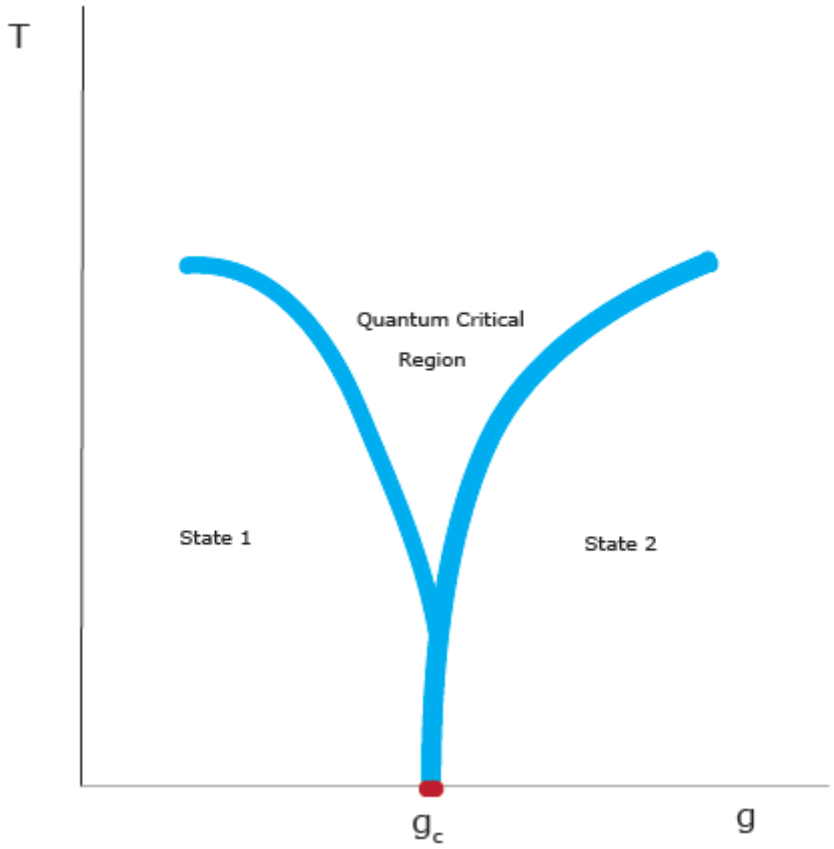

Figure 1: Shows the quantum critical region in systems such as $TlClCu_3$

coulomb repulsion is significant and needs to be taken into consideration. Such systems show quantum entanglement on a large scale and hence studying them using conventional techniques of condensed matter physics becomes hard.

On variation of an external parameter g, some of the strongly correlated materials can be tuned between two or more phases discussed above. Doping level, pressure applied on the solid or the value of external electric field are examples of an external parameter. The critical value of g which separates the two phases is called the quantum critical point. Temperature is not considered as the external parameter because we are interested in studying the ground state properties of the system. Moreover, strictly speaking quantum criticality is present only at absolute zero of temperature, but this does not make it a remote phenomenon [34, 32]. In fact, the effect of quantum criticality can be felt at temperatures higher than absolute zero thereby making it a phenomenon of general interest among the experimental as well as the theoretical physicists (Refer Fig.1).

The tuning parameter can be varied to achieve quantum criticality at $g = g_c$. The quantum transition can be of two types- first order and second order. The first order transitions are discontinuous, analogous to thermal phase transitions (eg. water boiling to steam at a certain pressure). But, the second order transitions are continuous and at the critical point the critical fluctuations are scale invariant. Therefore, for studying such systems it is necessary to study them using conformal field theory approach, as the conformal nature of the field theory ensures

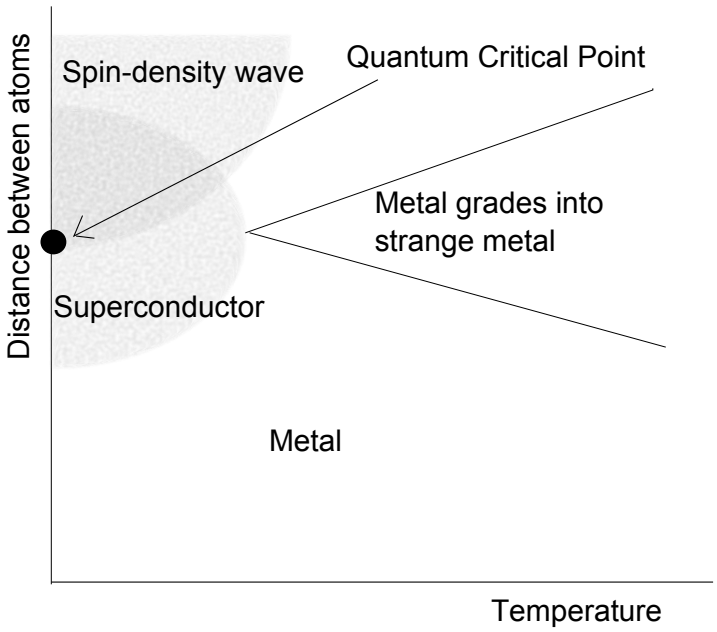

Figure 2: Shows different phases of electrons in matter

inherent scale invariance in the system. An example of application of QFT to condensed matter system (transport properties of graphene) is presented in [35].

Subir Sachdev and others have hypothesized that in most cases, high temperature superconductors happen to show "strange metal" behavior (Refer Fig 2). The 'strange metal' is a non-zero temperature phase of electrons in solids which appears to be a common to most strongly correlated compounds (esp. high temperature superconductors). It is defined as 'strange' because the temperature dependence of many observables deviate strongly from those expected from conventional Fermi liquid theory. The electrical resistance of strange metals increases linearly with temperature over a large range rather than with the square of the temperature as in normal metals [34, 32].

In [34] Subir Sachdev proposes that a quantum critical point may be an explanation for strange metallic behavior on the basis of the similarity between phase diagrams (Fig 1 and 2). Conformal field theories with Chern-Simons interactions represent a potentially interesting and partially unexplored class of quantum field theories that could arise at quantum critical points. Interactions represented by Chern-Simons theories are generically expected to arise, because of the planar nature of the quantum systems involved.

6 Conformal Field Theories in String theory and M-theory

Conformal field theories are very important in the formulation of string theory. They can be studied directly using field theory techniques (in a large $N-$ expansion), though this is often very difficult and only possible in limited cases. A new method of studying conformal field theories, discovered in string theory, is by a duality called the AdS/CFT duality. In this duality, physical quantities in the CFT can be calculated in an equivalent theory of gravity [36, 37]. One reason why this is possible is because the conformal group is the isometry group of the

Anti-deSitter space.

Using the AdS/CFT duality, one may, for instance, calculate the conductivity of a strongly interacting field theory by studying the absorption of electromagnetic waves in a black hole in a dual gravitational theory. The calculation in the gravity theory is usually fairly tractable in comparison to the calculation in the field theory which is impossible by conventional techniques. Classical theories of gravity thereby become models for strongly interacting quantum systems, including those that arise in condensed matter. This field is new and has attracted a lot of attention recently [38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 33].

The AdS/CFT correspondence makes the connection between string theory and large N gauge theories precise in certain highly supersymmetric cases. Before 2008, the primary example of the AdS/CFT correspondence was the $\mathcal{N} = 4$ $SU(N)$ super Yang Mills theory [49]. A new example of the AdS/CFT correspondence was discovered in 2008 [20], commonly known as ABJM. The theory is supersymmetric $U(N)_k \times U(N)_{-k}$ Chern-Simons theory coupled to matter transforming in a bifundamental representation, dual to M-theory on $AdS_4 \times S_7/Z_k$.

ABJM is also a theory of large $N \times N$ "matrices", and cannot be solved using traditional large N vector model techniques. However, ABJM may be easily generalized to an $U(M)_k \times U(N)_{-k}$ Chern-Simons theory. As it has been pointed out in [50], for $M = 1$ this theory is effectively a large-N vector model and hence exactly solvable in principle.

Moreover, as pointed out recently [51], one can study the ABJ theory (or similar theories with less or no supersymmetry) in the regime where M and N are both large, but $\frac{M}{N}$ is held fixed. A perturbative expansion in $\frac{M}{N}$ then allows us to approach the unsolvable large N saddle point with matrix degrees of freedom starting from the solvable large N saddle point with vector degrees of freedom. And, unlike the $1/N$ expansion, one may expect the $\frac{M}{N}$ expansion to converge, since $\frac{M}{N} = 1$ corresponds to a large N saddle point.

7 Research Objectives

1. To explore the scope of $\frac{M}{N}$ expansion as a new approach for the study of large-N quantum field theories.

2. To calculate anomalous dimensions in non-supersymmetric conformal Chern-Simons matter theories.

3. To calculate other physical quantities such as conductivity and free energy in Chern-Simons matter theories. This involves studying theories in presence of chemical potential for a conserved U(1) charge.

4. To interpret the results in context of dualities and compactifications of string theory/M theory.

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